Within the first few days of the Honors Algebra II course offered at the high school, you will be assessed on the prerequisite skills outlined in this packet. The packet will not be graded, however you are responsible for the material.

Please note that the assessment will count as a full test grade in your first quarter average.
Real Numbers and their Properties

Number Systems
- **Natural Number** (Counting Numbers) \(1, 2, 3, \ldots\)
- **Whole Numbers** (Introduce Zero) \(0, 1, 2, 3, \ldots\)
- **Integers** (Introduce Negative Numbers) \(\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\)

Note: The above number systems do not contain fractions or decimals.

- **Rational Numbers** (Numbers that can be written as the ratio of two integers.) When written as decimals, Rational Numbers terminate or repeat.
  
  Examples:
  \[
  \frac{2}{3} = 0.666\ldots \\
  \frac{3}{4} = 0.75 \\
  \frac{-5}{1} = -5
  \]

- **Irrational Numbers**
  Number that when written as decimals neither terminate nor repeat.

  Examples: \(\pi = 3.1415\ldots\) \(\sqrt{2} = 1.414213\ldots\)

- **Real Numbers** (All numbers on the number line.) This system combines the Natural #s, Whole #s, Integers, Rational #s, and Irrational #s.

Properties of Real Numbers

Let \(m, n, \) and \(p\) be real numbers.

<table>
<thead>
<tr>
<th>Property</th>
<th>Addition</th>
<th>Multiplication</th>
</tr>
</thead>
<tbody>
<tr>
<td>CLOSURE</td>
<td>(m + n) is a real number.</td>
<td>(mn) is a real number.</td>
</tr>
<tr>
<td>COMMUTATIVE</td>
<td>(m + n = n + m)</td>
<td>(mn = nm)</td>
</tr>
<tr>
<td>ASSOCIATIVE</td>
<td>((m + n) + p = m + (n + p))</td>
<td>((mn)p = m(np))</td>
</tr>
<tr>
<td>IDENTITY</td>
<td>(m + 0 = m, 0 + m = m)</td>
<td>(m \times (1) = m, 1 \times (m) = m)</td>
</tr>
<tr>
<td>INVERSE</td>
<td>(m + (-m) = 0)</td>
<td>(m \times \left(\frac{1}{m}\right) = 1, m \neq 0)</td>
</tr>
</tbody>
</table>
DISTRIBUTIVE (involves both addition and multiplication) $m(n + p) = mn + mp$

Additive Inverse – The opposite of any number $m$ is $-m$. 
Reciprocal – The multiplicative inverse of any nonzero number $m$ is $\frac{1}{m}$.

Exercises:

Identify the property being performed:

1) $8 \times 6 = 6 \times 8$
2) $4(6 + 3) = 4 \times 6 + 4 \times 3$
3) $7(1) = 7$
4) $-4 + 4 = 0$
5) $(9 \times 2) \times 3 = 9 \times (2 \times 3)$
6) $(4 + 5) + 3 = 4 + (5 + 3)$

Complete the chart by placing a check in the boxes that apply.

<table>
<thead>
<tr>
<th></th>
<th>Natural Numbers</th>
<th>Whole Numbers</th>
<th>Integers</th>
<th>Rational Numbers</th>
<th>Irrational Numbers</th>
<th>Real</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\frac{5}{9}$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\sqrt{2}$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Algebraic Expressions

The difference between an algebraic expression and an algebraic equation is that an expression does not have an equal to sign whereas an equation does.

Order of operations

- $P$ (Parentheses)
- $E$ (Exponents)
- $M$ & $D$ (Multiplication and Division are tied – perform operations from left to right)
- $A$ & $S$ (Addition and Subtraction are tied – perform operations from left to right)

Examples (Complete without the use of a calculator):
1) \(56 - 12 \div 3 \times 2\)  
Evaluate \(56 - 4 \times 2\)
\(56 - 8\)
\(48\)

2) \(-2(9 - 3^2) + 4^3 + 2\)

\(-2(9 - 9) + 64 \div 2\)
\(-2(0) + 32\)
\(0 + 32 = 32\)

---

**Exercises:**

Evaluate for the given value of \(x\):

1. \(x^3 \div 9 - 2x\) when \(x = -3\)

2. \(\left(\frac{3x^2 - 5x}{7x - 10}\right) \div 2\) when \(x = 5\)

Evaluate the expression without using a calculator:

3. \(16 \div \left(2\left(3^3 - 11\right) \div 4\right) + 5^2\)

4. \(\frac{2(5 - 7)^3}{1} + (15 \div 3 \times 2)\)

Simplify the expression:

5. \(-3(x^2 + 2x) - 5x(2x - 3)\)

---

**Solving Linear Equations**

Example with a variable on one side:

\(-\frac{2}{3}x + 5 = 13\)  
*subtract 5*

\(-\frac{2}{3}x = 8\)  
*multiply by the reciprocal of \(-\frac{2}{3}\)*

\(x = -12\)

\(-\frac{2}{3}(-12) + 5 = 13\)  
*Check your solution using substitution.*

\(13 = 13\)
Example with a variable on both sides:

1. \[6x - 13 = 22 - x\]  
   Move the smaller \(x\)-value. In this case, add \(x\) to both sides.

2. \[7x - 13 = 22\]  
   Add 13 to both sides.

3. \[7x = 35\]  
   Divide by 7.

4. \[x = 5\]

5. \[6(5) - 13 = 22 - 5\]  
   Check by substitution

17 = 17

Example that uses the distributive property:

1. \[-2(3x + 4) + 4x = -3(x + 2)\]  
   Use the distributive property.

2. \[-6x - 8 + 4x = -3x - 6\]  
   Combine like terms

3. \[-2x - 8 = -3x - 6\]  
   Move the smaller \(x\)-term. Add \(3x\) to both sides.

4. \[x - 8 = -6\]  
   Add 8 to both sides.

5. \[x = 2\]

6. \[-2(3(2) + 4) + 4(2) = -3(2 + 2)\]  
   Check by substitution.

7. \[-2(10) + 8 = -3(4)\]

8. \[-12 = -12\]

Exercises:

Solve the equation. Check the solution.

1. \[-3x + 14 = 11\]

2. \[\frac{1}{2}x - 8 = -3\]

3. \[4x - 12 = -3x + 9\]

4. \[\frac{2}{3}x - 2 = -\frac{3}{2}x - 4\]

5. \[6(-x - 5) = -4(x - 3) - x\]

6. Find the dimensions of the rectangle given the area = 504 sq. units.

\[
\begin{array}{c}
20x - 48 \\
\hline
7
\end{array}
\]
Rewriting Equations and Formulas
To solve for a particular variable means to isolate that variable.

Example: Solve $6x - 2y = 10$ for $y$.

$$-2y = -6x + 10 \quad \text{Subtract 6x}$$
$$y = 3x - 5 \quad \text{Divide by -2}$$

Example:
Given the equation $2x^2 - 3xy = 9$, find the value of $y$ when $x = -3$.

Method One: Solve for $y$ and then substitute.

$$2x^2 - 3xy = 9$$
$$-3xy = -2x^2 + 9$$
$$y = \frac{-2x^2 + 9}{-3x}$$
$$y = \frac{2}{3}x - \frac{3}{x}$$
$$y = \frac{2}{3}(-3) - \frac{3}{(-3)}$$
$$y = -1$$

Method Two: Substitute and then solve for $y$.

$$2x^2 - 3xy = 9$$
$$2(-3)^2 - 3(-3)y = 9$$
$$2(9) + 9y = 9$$
$$18 + 9y = 9$$
$$9y = -9$$
$$y = -1$$

Formulas:
- Temperature: $F = \frac{9}{5}C + 32$
- Area of a Triangle: $A = \frac{1}{2}bh$
- Area of a Rectangle: $A = bh$
- Perimeter of a Rectangle: $P = 2b + 2h$
- Area of a Trapezoid: $A = \frac{1}{2}h(b_1 + b_2)$
- Area of a Circle: $A = \pi r^2$
- Circumference of a Circle: $C = 2\pi r$ or $\pi d$
Example:
Solve for $h$ in the area of a trapezoid formula.

\[
A = \frac{1}{2} h (b_1 + b_2)
\]

\[
2A = h (b_1 + b_2)
\]

\[
\frac{2A}{(b_1 + b_2)} = h
\]

Exercises:

1. Solve for $y$:
   
   \begin{align*}
   a. \quad 2x - 3y &= 24; \quad x = 6 & b. \quad 2xy - 3y &= -91; \quad x = -5
   \end{align*}

2. Solve for the indicated variable:
   
   \begin{align*}
   a. \quad \text{Circumference of a circle} & \quad b. \quad \text{Temperature} \\
   \text{Solve for} \ d & \quad \text{Solve for} \ C
   \end{align*}

3. Write an equation to represent each scenario. Then solve.
   
   a. You are taking horseback riding lessons. The cost of the introductory lesson
   is $\frac{2}{3}$ the cost of each additional lesson. If you take a total of 6 lessons and
   spend a total of $340. How much was the introductory lesson?

   b. You purchase 3 bags of fertilizer for your vegetable garden. Each bag
   contains 50 cubic feet of fertilizer. If the fertilizer is spread a single foot
   thick and the length of the garden is 5 more than the width, what are the
   dimensions of the garden?

Solving Linear Inequalities

Inequality Symbols:

\begin{align*}
< & \quad \text{less than} & \quad \leq & \quad \text{less than or equal to} \\
> & \quad \text{greater than} & \quad \geq & \quad \text{greater than or equal to}
\end{align*}
Linear Inequalities in one variable are graphed on a number line. An open circle is used to represent the less than and the greater than symbols. A closed circle is used to represent the less than or equal to symbol and the greater than or equal to symbol. You then shade to either the left or right.

A compound inequality is two inequalities joined by either an “and” or an “or”.

When you multiply or divide by a negative number, the direction of the inequality sign switches.

Examples:

Solve: \(3x - 4 < 8\)

\(3x < 12\)

\(x < 4\)

4

Solve: \(-2x + 3 \geq -3x - 5\)

\(x + 3 \geq -5\)

\(x \geq -8\)

-8

Solve: \(-4 \leq -x + 6 < 10\)

\(-10 \leq -x < 4\)

\(10 \geq x > -4\)

-4, 10

Solve: \(x - 4 > -2\) or \(-x - 2 > 2\)

\(x > 2\) or \(-x > 4\)

\(x < -4\)

-4, 2

Exercises:
Solve and Graph:

1) \(-3x + 9 < 12\) 
2) \(7x - 10 \geq 11\) 
3) \(-\frac{2}{3}x - 4 \leq 8\)

4) \(-4 < 2x - 6 \leq 8\) 
5) \(-3x + 5 < -10\) or \(4x - 1 \leq 3\)

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Absolute Value Equations and Inequalities

**Absolute Value Sign with an Equal Sign:**

\[ |ax + b| = c \]

\[ \begin{align*}
ax + b &= c \\
ax + b &= -c
\end{align*} \]

Example:

\[ |2x - 5| = 11 \]

\[ \begin{align*}
2x - 5 &= 11 \\
2x &= 16 \\
x &= 8
\end{align*} \]

or

\[ \begin{align*}
2x - 5 &= -11 \\
2x &= -6 \\
x &= -3
\end{align*} \]

**Absolute Value Sign with a Less Than or a Less Than or Equal to Sign:**

\[ |ax + b| < c \quad |ax + b| \leq c \]

\[ \begin{align*}
-c < ax + b < c \\
-c \leq ax + b \leq c
\end{align*} \]

Example:

\[ |-x + 4| < 6 \]

\[ \begin{align*}
-6 < -x + 4 < 6 \\
-10 < -x < 2 \quad \text{Remember division by a neg. means switch the direction}
\end{align*} \]

\[ \begin{align*}
10 > x > -2
\end{align*} \]

**Absolute Value Sign with a Greater**

\[ |ax + b| > c \quad |ax + b| \geq c \]
Than or a Greater Than or Equal to Sign:

\[ ax + b > c \quad \text{or} \quad ax + b < -c \quad \text{or} \quad ax + b \geq c \quad \text{or} \quad ax + b \leq -c \]

Example:

\[
\begin{align*}
|4x - 6| & \geq 8 \\
4x - 6 & \geq 8 \\
4x & \geq 14 \\
x & \geq \frac{14}{4} \\
\text{or} \\
4x - 6 & \leq -8 \\
4x & \leq -2 \\
x & \leq -\frac{1}{2}
\end{align*}
\]

Exercises:

Solve:

1) \[ |4x - 6| \leq 10 \]
2) \[ \frac{1}{2} |5 - x| > 4 \]
3) \[ |8x - 2| = 14 \]
4) \[ |x - 8| + 4 < -12 \]
5) \[ 2 |7x + 9| - 5 = 55 \]

Functions

Domain: Set of all x-values (input values, independent variables).
Range: Set of all y-values (output values, dependent variables).
Function: There exists one and only one output value for each input value (no x’s repeat).

For Example: Is a function.
\[
\begin{array}{c|c|c|c|c}
X: & 2 & -2 & 3 & 4 \\
Y: & 0 & 4 & 4 & 5
\end{array}
\]
b/c no x’s (input values) repeat.

Vertical Line Test: If a vertical line intersects a graph at more than one point then the graph is NOT a function.

Coordinate Plane:

Ordered Pairs: \((x, y)\)
A Linear Function is of the form \( y = mx+b \), where \( m \) is the slope and \( b \) is the \( y \)-intercept. The degree of a linear function is one.

**Function notation:** \( f(x)=mx+b \)

**Exercises:**

1) Evaluate when \( x = -2 \). \( f(x) = -x^2 + x - 2 \)
2) Evaluate when \( x = 9 \). \( j(x) = x^3 - 2x^2 \)
3) Tell whether the relation is a function.
   Explain why or why not.

<table>
<thead>
<tr>
<th>( x )</th>
<th>0</th>
<th>0</th>
<th>2</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>-2</td>
<td>-1</td>
<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>

4) Identify the domain and range. Then state if it is a function.
5) Use the vertical line test to determine whether the relation is a function.

6) Graph:
   a) \( y = \frac{2}{3}x - 6 \)
   b) \( 2x - y = 8 \)
   c) \( x = -3 \)

7) State whether or not the function is linear.
   a) \( y = |2x - 3| + 1 \)
   b) \( y = -5 + 2x \)

8) Given: \( m = \frac{1}{2}g - 6 \), \( 0 \leq g \leq 10 \) Identify the domain and range of the relation.
**Slope**

**Slope:** Given \((x_1, y_1)(x_2, y_2)\) the slope is \(m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\text{rise}}{\text{run}}\)

Example: The slope of the line passing through the points \((4, -2) & (0, 3)\)

\[
= \frac{3 - (-2)}{0 - 4} = -\frac{5}{4}.
\]

---

**Parallel Lines:** Have the same slope.

**Perpendicular Lines:** Have opposite reciprocal slopes.

---

**Exercises:**

For questions 1 – 4, find the slope of the line passing through the given points. Then tell whether the line rises (+), falls(-), is horizontal (zero) or vertical (undefined).

1) \((-2, 3) \ (4, 5)\)  
2) \((4, -5) \ (4, 6)\)  
3) \((4, 9) \ (7, 5)\)  
4) \((-3, 6) \ (0,6)\)

For questions 5 – 6, tell whether the lines are parallel, perpendicular, or neither.

5) Line One: \((-2, 6) and (-2, 4)\)  
6) Line One: \((-2, 4) and (5, 6)\)
Graphing Linear Equations

Slope – Intercept Form: \( y = mx + b \)  
\( m \): slope \hspace{1cm} \( b \): y-intercept

Example: Graph \( y = -\frac{2}{3}x + 4 \)

\[ \text{STEP ONE: Plot } (0, 4). \]
\[ \text{STEP TWO: From that point, plot your second point by going down 2 and right 3.} \]
\[ \text{STEP THREE: Connect the two points.} \]

Standard Form: \( Ax + By = C \)  
Where \( A \) and \( B \) are not both zero.  
Solve by graphing both the x and y - intercepts.

Example: Graph \( 4x - 6y = -12 \)

\[ \text{STEP ONE: Set } x \text{ equal to zero and solve for } y \text{ (this is your y-intercept).} \]
\[ \text{STEP TWO: Set } y \text{ equal to zero and solve for } x \text{ (this is your x-intercept).} \]
\[ \text{STEP THREE: Plot both your x and y-intercepts. } (3, 0) \text{ and } (0, 2) \]
\[ \text{Connect the points.} \]

Horizontal Lines: Assumes the form \( y = c. \)

Vertical Lines: Assumes the form \( x = c. \)

Exercises:
1) State the slope and y-intercept of \( a. \ y = 2x + 4 \)
2. \( b. \ 4x - 6y = 3 \)
2) Find the intercepts of the line:
   a. $5x - 3y = 30$
   b. $y = 2x - 8$

3) Graph the equation:
   a. $y = \frac{-1}{3}x + 5$
   b. $-5x + 3y = 15$
   c. $y = 4$

4) Draw the line with the given slope and y-intercept.
   a. $m = -\frac{3}{4}$, $b = -2$
   b. $m = 0$, $b = -3$

---

**Additional Methods of Writing an Equation of a Line**

**Point – Slope Form:**

$y - y_1 = m(x - x_1) \\
(x_1, y_1)$: Point

$m$: slope

**Given Two Points:**

$(x_1, y_1)$: First Point

$(x_2, y_2)$: Second Point

**STEP ONE:** Calculate the slope.

**STEP TWO:** Substitute the slope found in step one and either the first or second point into $y = mx + b$. Then solve for $b$.

**STEP THREE:** You now have the slope and y-intercept ($b$). Substitute them into $y = mx + b$ and you are done.

**ALTERNATE STEPS TWO & THREE:** Substitute the slope found in step one and either the first or second point into $y - y_1 = m(x - x_1)$. Solve for $y$ in terms of $x$.

**Direct Variation:**

$y = kx$ or $k = \frac{y}{x}$.

$k$: constant of variation.

Use this formula when you are told two variables vary DIRECTLY rather than linearly.
Example: Given $x$ and $y$ vary directly, and $y = 8$ when $x = -2$.

| STEP ONE: Substitute known values into $y = kx$ | $8 = k(-2)$ |
| STEP TWO: Solve for $k$ | $k = -4$ |
| STEP THREE: Rewrite the equation using only $k$. | $y = -4x$ |

Exercises:

1) Write an equation of the line that adheres to the following criteria:

   a) Slope: $\frac{2}{3}$, $y$ -intercept: $-4$

   b) Slope: $-\frac{1}{4}$, though point $(4, -1)$.

   c) Passes through points $(1, 2)$ and $(4, 7)$.

   d) Passes through $(-2, 3)$ and is perpendicular to the line $y = -2x + 5$.

   e) Passes through $(3, -5)$ and is parallel to the line $y = \frac{2}{3}x - 6$.

   f) Passes through $(4, 6)$ and is parallel to the line that passes through $(4, -6)$ & $(8, -4)$.

2) $x$ and $y$ vary directly. Write an equation that relates the variables. Then find when $x = -6$. 
\[ x = -12 \text{ and } y = 2. \]

3) Tell whether the data show direct variation. If so, write the equation relating \( x \) and \( y \).

<table>
<thead>
<tr>
<th>( X )</th>
<th>-3</th>
<th>3</th>
<th>12</th>
<th>18</th>
</tr>
</thead>
<tbody>
<tr>
<td>( Y )</td>
<td>1</td>
<td>-1</td>
<td>-4</td>
<td>-6</td>
</tr>
</tbody>
</table>

4) The number of calories a person burns jogging varies directly with the time the person spends doing the activity. For example, when a 145 pound person jogs for 15 minutes he burns 145 calories. Write a linear model and then estimate how long a 145 pound person would have to jog to burn 400 calories.

5) In 1990 retail sales of music CDs were about $6.2 billion. In 1998 retail sales of music CDs were about $10.4 billion. Write a linear model for retail sales \( y \) (in billions of dollars) from 1990 to 1998. Let \( x \) represent the number of years since 1990. Then estimate the retail sales in 2010.

---

**Solving Linear Inequalities Algebraically and Graphically in Two Variables**

**Inequality Symbols:**

\( < \) less than \hspace{1cm} \( \leq \) less than or equal to

\( > \) greater than \hspace{1cm} \( \geq \) greater than or equal to

---

**Step 1:** Solve the inequality as though it were an equation. There is one exception! You must change the direction of the sign when you multiply or divide by a negative number.

**Step 2:** Graph the boundary line as you would the equation of a line. The boundary of a less than or a greater than symbol is represented by a broken line. The boundary line of a less than or equal to or a greater than or equal to symbol is represented by a solid line.

**Step 3:** Shade in the appropriate direction. Select a test point and substitute it into the inequality. If it becomes a true inequality, shade in the direction of that point. If it becomes a false inequality, shade away from that point.
Examples:
Graph: \(2x - y < 4\)

Graph: \(y \geq -\frac{1}{2}x + 3\)

Graph: \(x > -3\)

Exercises:
Graph the following inequalities on the coordinate plane:

1. \(y \leq -2x + 4\)
2. \(6x - 2y < 8\)
3. \(4y - x > 12\)
4. \(3x - 2y \geq 10\)

**Graphing Piecewise & Step Functions**

A piecewise function consists of a combination of equations, each restricted to a specified part of the domain.

Example: 
\[ f(x) = \begin{cases} 
  x + 4, & \text{if } x > 2 \\
-\frac{1}{2}x - 1, & \text{if } x \leq 2 
\end{cases} \]
Hint: Generate two tables, each pairing the desired domain with its respective equation. Then sketch each graph separately on the same coordinate plane.

<table>
<thead>
<tr>
<th>X</th>
<th>X + 4</th>
<th>Y</th>
<th>X</th>
<th>(-\frac{x}{2} - 1)</th>
<th>Y</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>2+4</td>
<td>6(open circle)</td>
<td>2</td>
<td>(-1/2) (2) -1</td>
<td>-2(closed circle)</td>
</tr>
<tr>
<td>3</td>
<td>3+4</td>
<td>7</td>
<td>0</td>
<td>(-1/2) (0) -1</td>
<td>-1</td>
</tr>
<tr>
<td>4 . .</td>
<td>4+4</td>
<td>8 . .</td>
<td>-2 . .</td>
<td>(-1/2) (-2) -1</td>
<td>0</td>
</tr>
</tbody>
</table>

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A step function is called such because it literally looks like the steps on a staircase. It is considered a piecewise function because it too consists of a combination of equations, each restricted to a specified part of the domain.

Example: \( f(x) = \begin{cases} 
-2, & \text{if } -2 \leq x < 0 \\
-1, & \text{if } 0 \leq x < 2 \\
0, & \text{if } 2 \leq x < 4 \\
1, & \text{if } 4 \leq x < 6 
\end{cases} \)

Hint: The above example is read \( y = -2 \) for \( x \) values between -2 and 0 (including 2), \( y = -1 \) for \( x \) values between 0 and 2 (including 0) etc.

Exercises:

\[ f(x) = \begin{cases} 
2x + 1, & \text{if } x > 1 \\
-2x, & \text{if } x \leq 1 
\end{cases} \]

For questions 1 and 2, use the piecewise function above to evaluate the given \( x \)-value.

1. \( f(1) \)
2. \( f(2) \)
Graph the function:

3. \( f(x) = \begin{cases} 
  x^2 - 3, & \text{if } x > 3 \\
  -2x + 2, & \text{if } x \leq 3 
\end{cases} \)

4. \( f(x) = \begin{cases} 
  -4, & \text{if } -3 \leq x < -1 \\
  -2, & \text{if } -1 \leq x < 2 \\
  0, & \text{if } 2 \leq x < 4 \\
  2, & \text{if } 4 \leq x < 6 
\end{cases} \)

**Absolute Value Functions**

The graph of an absolute value function resembles the letter “V”. The point where the graph changes direction is called the vertex.

The general form of an absolute value function is: \( y = a|x-h|+k \). Where \( h \) and \( k \) are the x and y coordinates of the vertex and \( a \) controls the “width” of the “V”. \( +a \) values produce an upright graph, while \( -a \) values invert the graph.

Example: \( y = 2|x+3|-4 \)

The vertex is at \((-3, -4)\). From that point you plot \( \text{UP 2 and over 1 both to the right and the left.} \)

Example: \( y = -\frac{2}{3}|x-2|+1 \)

The vertex is at \((2, 1)\). From that point you plot \( \text{DOWN 2 and over three both to the right and the left.} \)
Exercises:

Graph:

1. \( y = -2|x - 3| + 1 \)
2. \( y = \frac{1}{2}|x + 3| + 2 \)
3. \( y = -\frac{3}{2}|x - 4| + 5 \)

---

**Solutions to Exercises Page 2**

1. Commutative Property over Multiplication
2. Distributive Property
3. Multiplicative Identity
4. Additive Inverse
5. Associative Property over Multiplication
6. Associative Property over Addition

<table>
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<th></th>
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<th>Whole Numbers</th>
<th>Integers</th>
<th>Rational Numbers</th>
<th>Irrational Numbers</th>
<th>Real</th>
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<td></td>
<td></td>
<td>X</td>
<td>X</td>
<td></td>
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</tr>
<tr>
<td>5</td>
<td>X</td>
<td>X</td>
<td>X</td>
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<tr>
<td>( \frac{8}{9} )</td>
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<td></td>
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</tr>
</tbody>
</table>

**Solutions to Exercises Page 3**

1. 3
2. 1
3. 27
4. -70
5. \(-13x^2 + 9x\)
Solutions to Exercises Page 4

1. \( x=1 \)
2. \( x=10 \)
3. \( x=3 \)
4. \( x=-\frac{12}{13} \)
5. \( x=-42 \)
6. \( x=6 \) \( \therefore \) Dimensions are 7 x 72

Solutions to Exercises Page 6

1a. \( y=-4 \)   b. \( y=7 \)
2a. \( d = \frac{C}{\pi} \)   b. \( C = \frac{5}{9}(F-32) \)
3a. Let \( x \) = cost of a “regular” horseback riding lesson.
\[ \frac{2}{3}x + 5x = 340 \]
Price of a regular lesson: $60      Price of an introductory lesson: $40

b. Let \( x \) = the width of the garden
b. Let \( x+5 \) = the length of the garden
\[ V = (x)(5+x)(1)=150 \]
Dimensions: 10 x 15 x 1

Solutions to Exercises Page 7

1. \( x > -1 \)
2. \( x \geq 3 \)
3. \( x \geq -18 \)
4. $1 < x \leq 7$

5. $x > 5$ or $x \leq 1$

---

**Solutions to Exercises Page 9**

1. $-1 \leq x \leq 4$

2. $x < -3$ or $x > 13$

3. $x = 2$ or $x = -\frac{3}{2}$

4. Cannot be done. Absolute value cannot be less than or equal to a negative number.

5. $x = 3$ or $x = -\frac{40}{7}$

---

**Solutions to Exercises Page 10**

1. 0

2. 567

3. No. There exists two outputs for one input: $(0, -2)$ and $(0, -1)$

4. Domain: $\{-4, 2\}$ Range: $\{5, 6, 8\}$ No. There exists a two outputs for one input value. $(-4, 5)$ and $(-4, 6)$

5. No. It fails the vertical line test.

6a. $y = \frac{2}{3}x - 6$

   ![Graph 1](image1)

6b. $y = 2x - 8$

   ![Graph 2](image2)

6c. $x = -3$

   ![Graph 3](image3)

7a. No. It is an absolute value graph.

   b. Domain: $0 \leq x \leq 10$ Range: $-6 \leq y \leq -1$
Solutions to Exercises Page 11

1. \( \frac{1}{3} \) rises

2. Undefined vertical

3. \( -\frac{4}{3} \) falls

4. 0 horizontal

5. Line One: Undefined    Line Two: Zero    Perpendicular

6. Line One: \( \frac{2}{7} \)  Line Two: \( \frac{2}{7} \)    Parallel

7. Line One: \( \frac{9}{1} \)  Line Two: \( -3 \)    Line one is steeper.

Solutions to Exercises Pages 12 and 13

1a. \( m = 2 \)  \( b = 4 \)

b. \( m = \frac{2}{3} \)  \( b = -\frac{1}{2} \)

2a. (0, −10) (6, 0)

b. (0,−8) (4, 0)

3a. \( y = -\frac{1}{3}x + 5 \)

3b. 3b. \( y = \frac{5}{3}x + 5 \)

3c. \( y = 4 \)

4a. \( y = -\frac{3}{4}x - 2t \)

4b. \( y = -3 \)
Solutions to Exercises Pages 14 and 15

1a. \( y = \frac{2}{3} x - 4 \)

b. \( y = -\frac{1}{4} x \)

c. \( y = -x + 3 \)

d. \( y = \frac{1}{2} x + 4 \)

e. \( y = \frac{2}{3} x - 7 \)

f. \( y = \frac{1}{2} x + 4 \)

2. \( y = -\frac{1}{6} x; \ 1 \)

3. Yes it shows direct variation. \( y = -\frac{1}{3} x \)

4. Let \( C = \) Number of calories burned off \ Let \( A = \) Time spent on the activity

\[ C = \frac{29}{3} A; \ 41 \text{ minutes and approximately } 23 \text{ seconds} \]

5. \( y = \frac{21}{40} x + 6.2; \ 16.7 \text{ billion} \)
Solutions to Exercises Page 16

1. \( y \leq -2x + 4 \quad 2. \ y > 3x - 4 \quad 3. \ y > \frac{1}{4}x + 3 \)

Solutions to Exercises Page 17

1. -2
2. 5
3. 
4. 

Solutions to Exercises Page 18

1. 

---
2. [Shades][Poiz-Trace](?)

3. [Shades][Poiz-Trace](?)